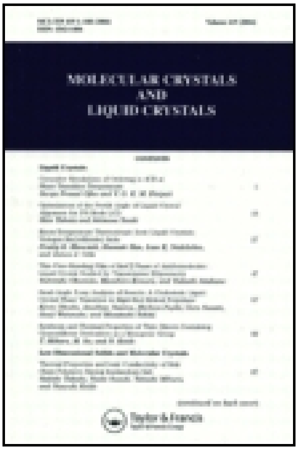

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Temperature as a Control Parameter of the Light Trajectories in Nematics with Topological Defects

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Temperature as a Control Parameter of the Light Trajectories in Nematics with Topological Defects

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The influence of controlable parameters like the temperature on the trajectories of light in a nematic liquid crystal with topological defects is studied through a geometric model. The model incorporates phenomenological details as how the refractive indices depend on such parameters. The deflection of light by the topological defect is then shown to be greater at lower temperatures.

Keywords: differential geometry; nematics; topological defects

PACS Numbers: 61.30.Jf – Defects in liquid crystals, 61.30.Dk – Continuum models and theories of liquid crystal structure and 42.70.Df – Liquid crystals

1. INTRODUCTION

Topological defects appear everywhere in physics. Ranging from gravitation and cosmology to Bose-Einstein condensates they are associated to symmetry-breaking phase transitions. In particular, they appear in the isotropic-to-nematic phase transition of calamitic liquid crystals in the form of hedgehogs, disclinations, domain walls or more complicated textures [1,2]. The deflection of light rays by these defects is an old issue. In this work we studied the propagation of light near disclination lines in nematics from a geometric point of view [3–5]. In particular, in [3] we observed lensing effects due

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the deflection of the beams in regions near the defect and calculated the light trajectories from a geometry resulting from the application of Fermat's principle associated to an effective refractive index N [6]. In a few words, what we did was to consider the bent light rays as geodesics of a model space of unknown geometry. By identifying Fermat's principle with the geodesic variational principle we were able to find the effective geometries for each defect studied. Knowing the effective geometry, the geodesics were obtained numerically.

Since the refractive indices, n_o and n_e, of a nematic depend both on the temperature and on the wavelenght of the light, a more realistic model incorporating these effects is in order. Li, Gauza, and Wu [7] modeled the temperature effect on the nematic refractive index based on Vuks equation and, by fitting their final expression to experimental data of selected materials, found the unknown coefficients. In this work, we incorporate their models to our geometric model for propagation of light in nematics with topological defects, in order to study temperature and wavelength effects. Although in [3] we studied both the symmetric ($k = 1$) and asymmetric ($k \neq 1$) defects, without loss of generality, we keep our analysis here mostly for the symmetric cases since their effective geometry is simpler and more intuitive than the asymmetric cases.

2. GEOMETRIC MODEL

Disclinations in nematics are classified according to the topological index (or strength) k which gives a measure of how much the director rotates as one goes around the defect. That is, the director configurations, in the plane x–y, are given by [1]

$$\varphi(\theta) = k\theta + c, \tag{1}$$

where φ is the angle between the molecular axis and the x-axis, θ is the angular polar coordinate and $c = \varphi(0)$. We consider an optical medium constituted by a nematic liquid crystal with disclinations [2], where the effective geometry for the light is defined by the line element [3]:

$$\begin{aligned} ds^2 = &\{n_o^2 \cos^2[(k-1)\phi + c] + n_e^2 \sin^2[(k-1)\phi + c]\}dr^2 \\ &+ \{n_o^2 \sin^2[(k-1)\phi + c] + n_e^2 \cos^2[(k-1)\phi + c]\}r^2 d\phi^2 \\ &- \{2(n_e^2 - n_o^2)^2 \sin[(k-1)\phi + c] \cos[(k-1)\phi + c]\}r dr d\phi. \end{aligned} \tag{2}$$

The metric (2) was obtained by identifying Fermat's principle with the variational principle that determines the geodesics in Riemannian geometry. Let

$$\mathcal{F} = \int_A^B N d\ell, \tag{3}$$

where, $d\ell$ is the element of arc length along the path between points A and B and the effective refractive index

$$N^2 = n_o^2 \cos^2 \beta + n_e^2 \sin^2 \beta, \tag{4}$$

where $\beta = \widehat{(\vec{n}, \vec{S})}$ is the local angle between the director $\vec{n}$ and the Poynting vector $\vec{S}$. In Riemannian geometry,

$$ds^2 = \sum_{i,j} g_{ij}\, dx^i dx^j, \tag{5}$$

where $g_{ij} = g_{ij}(x^{\mathrm{i}})$ is the metric tensor. The geodesic joinning points A and B in such manifold is obtained by minimizing $\int ds$, just like Fermat's principle. This leads to a nice interpretation of the light paths as geodesics in an effective geometry [6]. Thus, we may identify

$$N^2 d\ell^2 = \sum_{i,j} g_{ij} dx^i dx^j. \tag{6}$$

In [4] we showed that the effective geometry for the vortex-like $k = 1$, $c = \frac{\pi}{2}$ disclination is that of a cone. The effective metric for this case is obtained by substituting these values in metric (2) and rescaling the coordinate r to $\rho = n_e r$. That is, the two-dimensional line element for this effective geometry, in polar coordinates, is [3]

$$ds^2 = d\rho^2 + \alpha^2 \rho^2 d\theta^2, \tag{7}$$

$\alpha = n_o/n_e$ is the ratio between the refractive indices. The geodesic equation in a Riemannian space like the cone is [8]

$$\frac{d^2 x^i}{dt^2} + \sum_{j,k} \Gamma^i_{jk} \frac{dx^j}{dt} \frac{dx^k}{dt} = 0, \tag{8}$$

where t is a parameter along the geodesic and Γ^i_{jk} are the Christoffel symbols, given by

$$\Gamma^i_{jk} = \frac{1}{2} \sum_m g^{mi} \left\{ \frac{\partial g_{km}}{\partial x^j} + \frac{\partial g_{mj}}{\partial x^k} - \frac{\partial g_{jk}}{\partial x^m} \right\}. \tag{9}$$

For metric (7) (conic space), the Eq. (8) reduces to a coupled system of ordinary differential equations with solution given in [9].

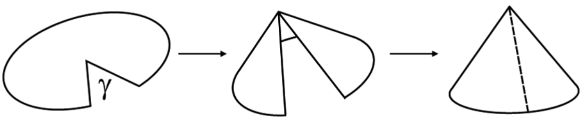

FIGURE 1 Conical surface of angular deficit γ.

The Figure 1 shows the making of a cone from a planar sheet where an angular section was removed with posterior identification of the edges. If γ is the angle that defines the removed section then the remaining surface corresponds to an angular sector of $2\pi\alpha = 2\pi - \gamma$. This is exactly what metric (7) describes. The incorporation of the term α^2 to the Euclidean metric in polar coordinates makes the total angle on the surface be $\int_0^{2\pi} \alpha d\theta = 2\pi\alpha < 2\pi$, since $n_o < n_e$. It is clear then that α tells how "pointed" is the cone. The closer α gets to 1 the flatter is the cone. For $\alpha = 1$ the cone turns into a plane.

3. REFRACTIVE INDEX VARIATION

The refractive indices n_o and n_e of a nematic liquid crystal depend on the temperature (T). Based in [7], we analyse how this parameter affect the ratio $\alpha = n_o/n_e$, which characterizes the effective geometry associated to disclinations. By changing either T, α is changed and so is the effective geometry. This causes a deformation of the geodesics associated to light rays in our model.

In [7] we can find expressions to the ordinary and extraordinary refractive index given in terms of the birefringence $\Delta\ n$ and of its average value $\langle n \rangle$, such that

$$n_o = \langle n \rangle - \frac{1}{3}\Delta n, \tag{10}$$

$$n_e = \langle n \rangle + \frac{2}{3}\Delta n.$$

In (10) and (11) the behavior of $\langle n \rangle$ as fuction of the temperature [7] is given experimentally through a linear dependence given by

$$\langle n \rangle = A - BT, \tag{12}$$

where the parameters A and B are obtained experimentally.

The birefringence can be written in terms of the approximated [10] order parameter $S = \left(1 - \frac{T}{T_c}\right)^{\beta}$ as

$$\Delta n = (\Delta n)_0 \left(1 - \frac{T}{T_c}\right)^{\beta}, \tag{13}$$

where $(\Delta n)_0$ is the birefringence in the crystalline state, β is a constant associated to the material and T_c is he isotropic-nematic transition temperature.

Therefore, substituting the Eqs. (12) and (13) into (10) and (11), we have

$$n_o = A - BT - \frac{(\Delta n)_0}{3}\left(1 - \frac{T}{T_c}\right)^{\beta}, \tag{14}$$

$$n_e = A - BT + \frac{2(\Delta n)_0}{3}\left(1 - \frac{T}{T_c}\right)^{\beta}. \tag{15}$$

The liquid crystal considered was the 5CB and the wavelength of the incident beam was 589 nm [7]. For this material the parameters are given in the table below obtained from [7]. The parameters A, B, β and $(\Delta n)_0$ are adimensional.

A	B	β	$(\Delta n)_0$	T_c
1.7546	0.0005360 K^{-1}	0.2391	0.3768	306.6 K

The temperature causes α to change and we can summarize its effect on the light paths by studying the geodesics for different values of α (Fig. 2). Substituting the metric (2) in (9) and this one in (8) we can calculate the geodesics for different values of α. As described, the

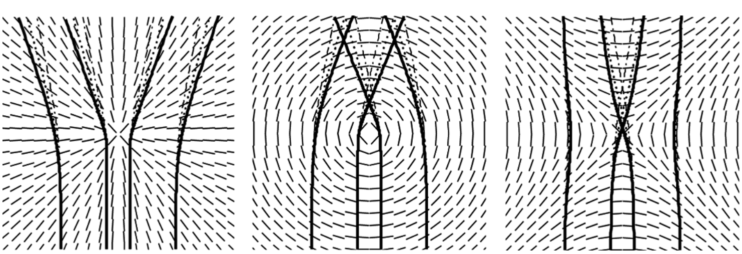

FIGURE 2 Influence of the parameter α on the light trajectories in a nematic liquid crystal with a disclination $k = 1$ and $c = 0$.

geodesic equations (8) have exact solutions for the $k = 1$ case. The remaining cases can be solved by a numerical method. Below we show the effects of the variation of the parameter α on the light paths near the $k = 1$ defects, using the exact solution and same effects for the $k = -1$ disclination, using the Runge-Kutta numerical method to solve the geodesic equation. In all cases, the solid line corresponds to $\alpha = 0{,}8912$ ($T = 290\,K$), the dotted line to $\alpha = 0{,}9120$ ($T = 300K$) and the dash-dotted line to $\alpha = 0{,}9355$ ($T = 305K$). Notice that as α approaches 1 the light paths straighten out, as it should.

4. CONCLUSION

We associated the light paths to geodesics in a curved space specified by a topological defect (defects in nematics cause light passing by to deflect). The deflection is due to the particular orientation of the director field associated to the defect, which may be translated into curvature. The intensity of the deflection depends on the ratio α between the ordinary and extraordinary refractive indices, which, in turn, depends on the temperature of the liquid crystal. Taking as example 5CB, which has been extensively characterized with respect to temperature dependence of the refractive indices [7], we solved the geodesic equations for a realistic range of values of α corresponding to temperature variation. The graphical result illustrates the influence of this parameter on the light deflection caused by the defect. The further α gets from 1 the strong is the deflection. This can be achieved by either lowering the temperature.

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